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Three-dimensional analysis of reinforced concrete frames based on lumped damage mechanics

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Abstract

This paper presents a general formulation for the analysis of reinforced concrete frames. The model has been developed within the framework of lumped damage mechanics. This is a theory based on the methods of continuum damage mechanics, fracture mechanics and the concept of plastic hinge. The paper also describes the numerical implementation of the model in the finite element programs. The model is evaluated by the numerical simulation of three tests reported in the literature. Two of them deal with a column subjected to variable axial loads and biaxial flexure. The third is a two-story three-dimensional frame subjected to earthquake loadings outside the principal directions of the frame.

Keywords: Fracture mechanics; Damage mechanics; Structural analysis; Biaxial bending; Reinforced concrete

1. Introduction

The inelastic analysis of RC framed structures can be carried out in two ways. The first approach, which is called the fiber beam theory, represents the cross section of each frame member as a set of small filaments with finite length and in series along the element. Each filament is characterized by the uniaxial constitutive law that represents the behavior of concrete or steel. Oliva and Clough (1987) list the following references for the fiber beam theory: Aktan et al. (1973), Okada et al. (1976), Takizawa and Aoyama (1976). The models proposed by Roufaiel and Meyer (1987), Zeris and Mahin (1991), Anthoine et al. (1997), Oller et al. (1992) and Bahn and Hsu (2000) can also be included in this category. For very large three-dimensional structures, the fiber beam theory is computationally expensive. The second approach, which is called the frame theory, is based on the formulation of interaction plasticity surfaces and flow rules to define the flexure-rotational behavior of the element (Nigam, 1967; Padilla-Mora and Schnobrich, 1974; Selna and Lawder, 1977; Chen and Powell, 1982; Lai et al., 1984). The frame theory is computationally much cheaper

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than the fiber beam theory. The model in the present paper belongs to the frame theory. While all the previous models in the frame theory have used the lumped plasticity, we propose a new model based on the lumped damage mechanics as introduced below.

The basic idea in the lumped damage mechanics is the combination of the methods of continuum damage and fracture mechanics with the concept of plastic hinge. It provides a general framework for the analysis of framed structures under severe overloads (typically earthquake loadings), high cycle fatigue, impacts or blasts. It can be widely applied to civil engineering structures (buildings, bridges) and some offshore and industrial structures. Cipollina et al. (1995), Bolzon (1996), Flórez-López (1998), Mazza (1998), Perdomo et al. (1999) and Perera et al. (2000) have used the lumped damage mechanics to planar frames. The present work and a previous one by the authors (Marante and Flórez-López, 2002) attempt to deal with three-dimensional space frames. A model that describes the process of damage due to biaxial flexure is presented by taking into account the possibility of variable axial forces and torques. The paper also describes the finite element implementation of the model. Three test results reported in the literature (Bousias et al., 1995; Oliva, 1980; Oliva and Clough, 1987) are simulated to validate the model.

2. Kinematics of spatial frames

Consider a special frame with m members connected by n nodes. The movement of the structure during the time interval $[0, T]$ is analyzed. A set of orthogonal coordinate axes X, Y and Z is introduced in order to define the position of each node at any configuration. The generalized displacements of a node i are denoted by $\{\mathbf{u}\}_i^{\mathrm{t}} = (u_1, u_2, \ldots, u_6)$ (t indicates "transpose"), where u_1, u_2 and u_3 are the displacements in the X, Y and Z directions, respectively, and u_4, u_5 and u_6 indicate the corresponding rotations. The displacement matrix of the frame is given by $\{\mathbf{U}\}^{\mathbf{t}} = (\{\mathbf{u}\}_1, \{\mathbf{u}\}_2, \ldots, \{\mathbf{u}\}_n)$.

Consider a frame member b with the end nodes i and j. A local set of orthogonal coordinate axes x, y and z is introduced. The direction x coincides with the neutral axis of the frame member and y and z are the principal directions of the cross section. The generalized strain, or deformation, matrix is given by $\Phi_b^{\mathrm{t}} = (\phi_{iy}, \phi_{jy}, \delta, \phi_{iz}, \phi_{jz}, \phi_x)$, where δ represents the elongation of the chord and ϕ_x is the angle of twist as seen in Fig. 1. The components ϕ_{iy} and ϕ_{jy} are the flexural rotations of the tangents to the member with respect to the chord $i - j$, in the xz plane, while ϕ_{iz} and ϕ_{jz} are the flexural rotations in the xy plane. The relationship between member strains and nodal displacements can be written as follows:

$$\{\dot{\mathbf{\Phi}}\}_b = [\mathbf{B}(\mathbf{U})]_b\{\dot{\mathbf{U}}\} \quad \text{or} \quad \{\mathbf{\Phi}\}_b = \int_0^{\mathrm{t}} [\mathbf{B}(\mathbf{U})]_b\{\dot{\mathbf{U}}\}\,\mathrm{d}\tau, \tag{1}$$

where the transformation matrix **[B]** is a function of the nodal displacements when geometrically nonlinear effects are taken into account. The nonzero part of **[B]** is given by

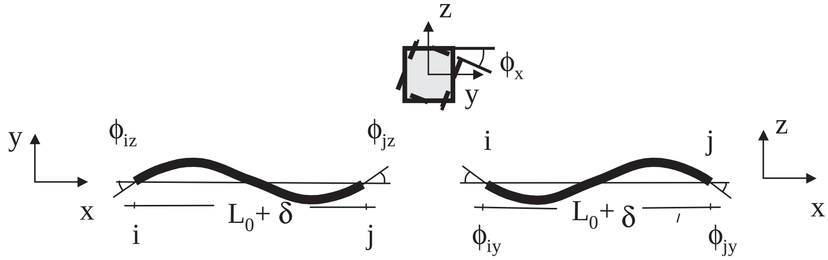

Fig. 1. Generalized strains of a member between nodes i and j.

$$[\mathbf{B}] = \begin{bmatrix} -\frac{m_1}{L} & -\frac{m_2}{L} & -\frac{m_3}{L} & n_1 & n_2 & n_3 & \frac{m_1}{L} & \frac{m_2}{L} & \frac{m_3}{L} & 0 & 0 & 0 \\ -\frac{m_1}{L} & -\frac{m_2}{L} & -\frac{m_3}{L} & 0 & 0 & 0 & \frac{m_1}{L} & \frac{m_2}{L} & \frac{m_3}{L} & n_1 & n_2 & n_3 \\ -t_1 & -t_2 & -t_3 & 0 & 0 & 0 & t_1 & t_2 & t_3 & 0 & 0 & 0 \\ \frac{n_1}{L} & \frac{n_2}{L} & \frac{n_3}{L} & m_1 & m_2 & m_3 & -\frac{n_1}{L} & -\frac{n_2}{L} & -\frac{n_3}{L} & 0 & 0 & 0 \\ \frac{n_1}{L} & \frac{n_2}{L} & \frac{n_3}{L} & 0 & 0 & 0 & -\frac{n_1}{L} & -\frac{n_2}{L} & -\frac{n_3}{L} & m_1 & m_2 & m_3 \\ 0 & 0 & 0 & -t_1 & -t_2 & -t_3 & 0 & 0 & 0 & t_1 & t_2 & t_3 \end{bmatrix}, \tag{2}$$

where **t**, **n**, and **m** are the unit vectors in the x, y, and z directions, respectively. Note that the zeros must be added to the columns and the lines that do not correspond to any of the degrees of freedom of the element. The components of these vectors are expressed with respect to the global system of reference.

3. Dynamics of spatial frames

The equilibrium equation for the spatial frames can be obtained via the principle of virtual power,

$$P_{\mathrm{i}}^* + P_{\mathrm{a}}^* = P_{\mathrm{e}}^* \quad \forall\{\mathbf{U}^*\}, \tag{3}$$

where P_{i}^* is the deformation power (or internal power), P_{a}^* the inertial forces power and P_{e}^* the external nodal forces power. The deformation power is obtained by the introduction of the generalized stresses vector for a frame member $\{\mathbf{M}\}_b^{\mathrm{t}} = (m_{iy}, m_{jy}, n, m_{iz}, m_{jz}, m_x)$, which is conjugate to the member deformation measure. Notice that n and m_x are the axial force and the torque and m_{iy} and m_{jy} are flexural moments in the xz plane, m_{iz} and m_{jz} in the xy plane as shown in Fig. 2.

The deformation power is then given by

$$P_{\mathrm{i}}^* = \sum_{b=1}^{m} \{\dot{\mathbf{\Phi}}^*\}_b^{\mathrm{t}} \{\mathbf{M}\}_b = \{\dot{\mathbf{U}}^*\}^{\mathrm{t}} \sum_{b=1}^{m} [\mathbf{B}]_b^{\mathrm{t}} \{\mathbf{M}\}_b, \tag{4}$$

where $\{\dot{\mathbf{\Phi}}^*\}_b$ and $\{\dot{\mathbf{U}}^*\}$ the virtual strain and displacement rate vectors.

The power of the inertial forces is obtained by the introduction of the mass matrix of the element and is given by

$$P_{\mathrm{a}}^* = \sum_{b=1}^{m} \{\dot{\mathbf{U}}^*\}^{\mathrm{t}} [\mathbf{mass}]_b \{\ddot{\mathbf{U}}\} \tag{5}$$

and the power of the nodal forces by

$$P_{\mathrm{e}}^* = \{\dot{\mathbf{U}}^*\}^{\mathrm{t}} \{\mathbf{P}\}, \tag{6}$$

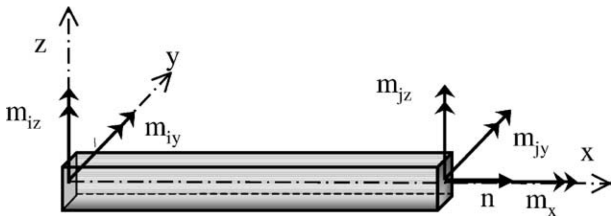

Fig. 2. Generalized stresses in a frame member.

where $\{\mathbf{P}\}$ is the nodal external force vector. The principle of virtual power for a framed structure results in

$$\{\dot{\mathbf{U}}^*\}^{\mathrm{t}}\sum_{b=1}^{m}[\mathbf{B}]_b^{\mathrm{t}}\{\mathbf{M}\}_b+\{\dot{\mathbf{U}}^*\}^{\mathrm{t}}\sum_{b=1}^{m}[\mathbf{mass}]_b\{\ddot{\mathbf{U}}\}=\{\dot{\mathbf{U}}^*\}^{\mathrm{t}}\{\mathbf{P}\},\quad\forall\{\dot{\mathbf{U}}^*\}. \tag{7}$$

Eliminate the virtual displacement rate vector in (7) to obtain the following equilibrium equation:

$$\sum_{b=1}^{m}[\mathbf{B}]_b^{\mathrm{t}}\{\mathbf{M}\}_b+\sum_{b=1}^{m}[\mathbf{mass}]_b\{\ddot{\mathbf{U}}\}=\{\mathbf{P}\}. \tag{8}$$

4. Lumped damage mechanics

4.1. Complementary elastic energy and state laws

A generalized constitutive law for a frame member can be obtained by using the lumped dissipation hypothesis. Thus, a frame member is assumed to be the assemblage of an elastic beam-column and two inelastic hinges as is shown in Fig. 3. All energy dissipation phenomena are assumed to be lumped at the inelastic hinges. In order to describe these effects, three sets of internal variables are now introduced. The first one corresponds to the plastic strains matrix: $\{\mathbf{\Phi}_p^{\mathrm{t}}\}=(\phi_{iy}^p,\phi_{jy}^p,\delta^p,\phi_{iz}^p,\phi_{jz}^p,\phi_x^p)$.

Plastic rotations in RC frame members are mainly the consequence of the reinforcement yielding, while the inelastic phenomena associated to concrete cracking are represented by the damage variables that are introduced as in Marante and Flórez-López (2002): $\{\mathbf{D}^+\}=(d_{iy}^+,d_{jy}^+,d_{iz}^+,d_{jz}^+)$ and $\{\mathbf{D}^-\}=(d_{iy}^-,d_{jy}^-,d_{iz}^-,d_{jz}^-)$. These variables describe damage due to flexural effects. The damage parameters can take values between zero and one, where zero represents a non-damaged hinge and one a totally damaged hinge with no stiffness at all, i.e. a totally damaged hinge behaves as internal hinges in elastic frames. The damage parameters with the superscript + (respectively −) represent damage, i.e. concrete cracking, due to positive (negative) moments such as indicated in Fig. 4. Parameters with subscripts iy characterize the damage due to the moment m_{iy} and so on.

The elasticity law of a frame member with damaged plastic hinges can be expressed as:

$$\{\mathbf{\Phi}-\mathbf{\Phi}_p\}=[\mathbf{F}(\mathbf{D}^+)]\langle\mathbf{M}\rangle_+ +[\mathbf{F}(\mathbf{D}^-)]\langle\mathbf{M}\rangle_-. \tag{9}$$

The terms $[\mathbf{F}(\mathbf{D})]$ represent the flexibility matrices that were proposed in Marante and Flórez-López (2002). The symbols $\langle m\rangle_+$ and $\langle m\rangle_-$ are the positive and negative parts of the variable m, i.e.:

$$\langle m\rangle_+=\begin{cases}m & \text{if } m\geqslant 0,\\ 0 & \text{otherwise,}\end{cases}\qquad \langle m\rangle_-=\begin{cases}m & \text{if } m\leqslant 0,\\ 0 & \text{otherwise.}\end{cases} \tag{10}$$

It can be seen that for positive moments, the flexibility matrix depends only on the positive damage and vice versa. In this way, the crack closure effects are represented in a simple manner. In continuum damage mechanics, a state law such as (9) is denoted "unilateral". A justification of this equation can be seen in Flórez-López (1998). The state law (9) could also include a term of "initial strains or stresses". In the frame

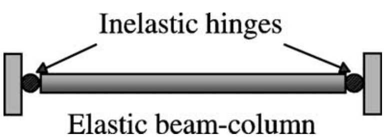

Fig. 3. Inelastic hinges in a frame member.

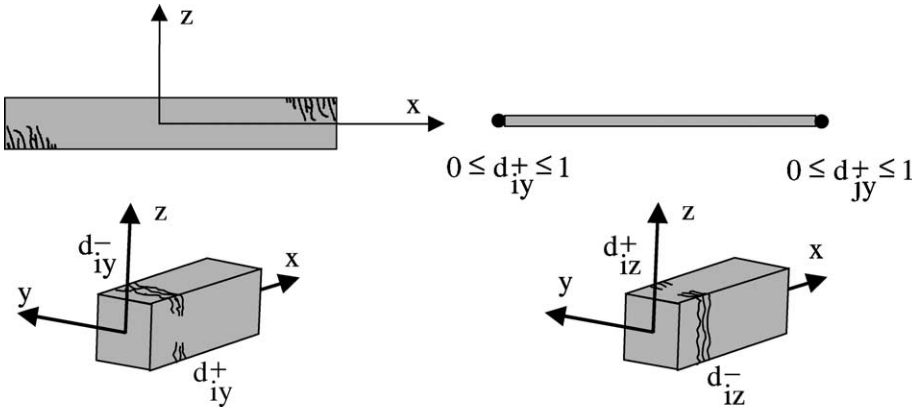

Fig. 4. Representation of cracking in a RC frame member via damage variables.

context, these stresses represent the so called "fixed end moments" due to external distributed forces applied on the structural member. In most practical applications these forces are not the cause of damage or yielding and can be taken into account as nodal forces. The complementary elastic energy W^* is now defined as follows:

$$W^* = \tfrac{1}{2}\{\mathbf{M}\}^{\mathrm{t}}\{\mathbf{\Phi} - \mathbf{\Phi}^p\} = \tfrac{1}{2}\{\mathbf{M}\}^{\mathrm{t}}[\mathbf{F}(\mathbf{D}^+)]\langle\mathbf{M}\rangle_+ + \tfrac{1}{2}\{\mathbf{M}\}^{\mathrm{t}}[\mathbf{F}(\mathbf{D}^-)]\langle\mathbf{M}\rangle_-. \tag{11}$$

Two matrices of energy release rates can be defined from the thermodynamic potential W^*: $\{\mathbf{G}^+\} = (G^+_{iy}, G^+_{jy}, G^+_{iz}, G^+_{jz})$, $\{\mathbf{G}^-\} = (G^-_{iy}, G^-_{jy}, G^-_{iz}, G^-_{jz})$, where:

$$G^+_{iy} = -\frac{\partial W^*}{\partial d^+_{iy}} = \frac{F^0_{11}}{2}\frac{\langle m_{iy}\rangle^2_+}{(1-d^+_{iy})^2}; \quad G^-_{iy} = -\frac{\partial W^*}{\partial d^-_{iy}} = \frac{F^0_{11}}{2}\frac{\langle m_{iy}\rangle^2_-}{(1-d^-_{iy})^2}; \quad \text{and so on.} \tag{12}$$

4.2. Damage evolution law

Marante and Flórez-López (2002) proposed a Rankine-type criterion of damage as a function of the energy release rates of a hinge i. The criterion is represented by the "non-damage convex" and the normality law that are shown in Fig. 5.

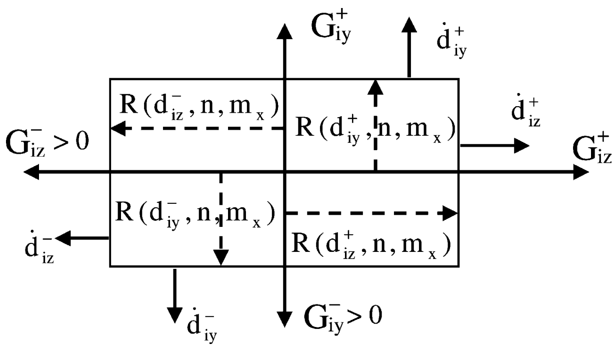

Fig. 5. Damage criterion in the energy release rate space.

Therefore, the "non-damage zone" is limited by the lines:

$$\begin{aligned} G_{iy}^{+} &= R_{iy}^{+}(d_{iy}^{+}; n, m_x); \quad G_{iy}^{-} = R_{iy}^{-}(d_{iy}^{-}; n, m_x); \\ G_{iz}^{+} &= R_{iz}^{+}(d_{iz}^{+}; n, m_x); \quad G_{iz}^{-} = R_{iz}^{-}(d_{iz}^{-}; n, m_x). \end{aligned} \tag{13}$$

The functions R are denoted crack resistance functions of the inelastic hinge i. It can be noticed that in the particular case of loadings in only one plane, the damage criterion becomes a generalized form of the Griffith criterion for a plastic hinge. Crack resistance functions have been identified from experimental results and depend on the axial force, the torque and the corresponding damage variable. An expression of R derived from the one proposed in Cipollina et al. (1995) is

$$R = G_{\rm cr}(n, m_x) + q(n, m_x)\frac{\ln(1-d)}{1-d}. \tag{14}$$

The term $G_{\rm cr}$ represents the crack resistance of the plastic hinge when there is no cracking ($d = 0$). This value can be expressed as a function of the first cracking moment of the cross section $m_{\rm cr}$:

$$G_{\rm cr} = \tfrac{1}{2}F_0 m_{\rm cr}^2(n, m_x). \tag{15}$$

The first cracking moment can be computed by the standard methods of the classic reinforced concrete theory. Notice that the cracking moment depends on the level of axial force and torque, therefore the term $G_{\rm cr}$ does too.

The other parameter of the crack resistance, the term $q(n, m_x)$ can also be computed in a similar way, but this time as function of the ultimate moment of the cross section $m_{\rm u}$. Indeed, the expression $G = R$ defines a relationship between the moment and the damage:

$$\frac{F_0 m^2}{2} = (1-d)^2 G_{\rm cr} + q(1-d)\ln(1-d). \tag{16}$$

With the help of the expression (16), the ultimate moment $m_{\rm u}$ can be related with a damage value $d_{\rm u}$. It can be noticed again that the ultimate moment also depends on the axial force and the torque. For these values the function $F_0 m^2/2$ reaches a maximum:

$$\begin{aligned} \frac{F_0 m_{\rm u}^2(n, m_x)}{2} &= (1-d_{\rm u})^2 G_{\rm cr} + q(1-d_{\rm u})\ln(1-d_{\rm u}) \\ &\quad - 2(1-d_{\rm u})G_{\rm cr} + q[\ln(1-d_{\rm u}) + 1] = 0. \end{aligned} \tag{17}$$

From Eq. (17), the value of q (and that of $d_{\rm u}$) can be computed. Again, the ultimate moment can be computed via the classic theory of reinforced concrete structures.

4.3. Yield function

The plastic behavior of a damaged plastic hinge can be obtained by the introduction of an "effective moment on a plastic hinge" and the strain equivalence hypothesis. By analogy with the effective stress of continuum damage mechanics, the effective moments $\overline{m}_{iy}$ and $\overline{m}_{iz}$ on a plastic hinge i are introduced as follows:

$$\overline{m}_{iy} = \begin{cases} \dfrac{m_{iy}}{1-d_{iy}^{+}} & \text{if } d_{iy}^{+} \text{ is active,} \\ \dfrac{m_{iy}}{1-d_{iy}^{-}} & \text{if } d_{iy}^{-} \text{ is active,} \end{cases} \qquad \overline{m}_{iz} = \begin{cases} \dfrac{m_{iz}}{1-d_{iz}^{+}} & \text{if } d_{iz}^{+} \text{ is active,} \\ \dfrac{m_{iz}}{1-d_{iz}^{-}} & \text{if } d_{iz}^{-} \text{ is active.} \end{cases} \tag{18}$$

Now, the yield function of a plastic hinge with damage can be obtained from any of the expressions proposed in the literature for reinforced concrete frame members by substitution of the moments with the effective moments. The plastic strains evolution law can be obtained via normality rule:

$$f_i = f_i(\overline{m}_{iy}, \overline{m}_{iz}, m_x, n, \phi^p_{iy}, \phi^p_{iz}, \phi^p_{ix}, \delta^p_i); \quad \dot{\phi}^p_{iy} = \dot{\lambda}_i \frac{\partial f_i}{\partial m_{iy}}; \quad \dot{\phi}^p_{iz} = \dot{\lambda}_i \frac{\partial f_i}{\partial m_{iz}}; \quad \dot{\phi}^p_{ix} = \dot{\lambda}_i \frac{\partial f_i}{\partial m_x}; \quad \dot{\delta}^p_i = \dot{\lambda}_i \frac{\partial f_i}{\partial n}, \tag{19}$$

the plastic axial deformation and twist of the frame member, are obtained by adding the contribution of both hinges:

$$\dot{\phi}^p_x = \dot{\lambda}_i \frac{\partial f_i}{\partial m_x} + \dot{\lambda}_j \frac{\partial f_j}{\partial m_x}; \quad \dot{\delta}^p = \dot{\lambda}_i \frac{\partial f_i}{\partial n} + \dot{\lambda}_j \frac{\partial f_j}{\partial n}. \tag{20}$$

In Marante and Flórez-López (2002), a classical criterion, the Bresler interaction function (Bresler, 1960), was chosen as a point of departure. A more general criterion (the Bresler function does not consider plastic twist) could also be used.

5. Time-discrete solution algorithm

The strain–displacement equation (1) and the constitutive law (9)–(20) define a relationship between the generalized stresses in an element b and the nodal displacements: $\{\mathbf{M}\}_b = \{\mathbf{M}(\mathbf{U})\}_b$. The nodal accelerations can also be expressed as a function of the nodal displacements, after time discretization by finite differences and the use of the Newmark method. Therefore, the time interval $[0, T]$ is substituted by a discrete set of instants $(0, t_1, t_2, \ldots, T)$. The frame is analyzed only for these times by using a conventional step-by-step method. The difference between two consecutive instants $(\Delta t = t_s - t_{s-1})$ is called "global step". Then, the equilibrium equation (8) at a given time t_s can be written as:

$$\{\mathbf{L}(\mathbf{U})\} = \sum_{b=1}^{m} [\mathbf{B}]^{\mathrm{t}}_b \{\mathbf{M}(\mathbf{U})\}_b + [\mathbf{mass}]\{\ddot{\mathbf{U}}(\mathbf{U})\} - \{\mathbf{P}\} = 0. \tag{21}$$

This equation, with the boundary conditions, defines the so called "global problem".

The global problem is solved via the Newton–Raphson method. Each iteration of the global problem for a given time t_s requires the solution of the following linear problem:

$$\{\mathbf{L}(\mathbf{U})\} \cong \{\mathbf{L}(\mathbf{U_0})\} + \left[\frac{\partial \mathbf{L}}{\partial \mathbf{U}}\right]_{\{\mathbf{U}\}=\{\mathbf{U_0}\}} \{\mathbf{U} - \mathbf{U_0}\} = 0, \tag{22}$$

where $\{\mathbf{U_0}\}$ represents the displacement matrix at the time t_n obtained during the precedent iteration and $\{\mathbf{U}\}$ is the displacement at the present iteration. It can be noticed that in order to build the residual matrix $\{\mathbf{L}(\mathbf{U_0})\}$ and its Jacobian, the computation of all the matrices $\{\mathbf{M}(\mathbf{U_0})\}_b$ and its derivatives is needed. This calculation is called "local problem". This problem is in general non-linear and requires the use of the Newton–Raphson method again.

The convergence requirements of the m local problems and the global problem are very diverse. Usually, most of the local problems are elastic or have little nonlinearity. Damage concentrates in the remaining elements that become highly nonlinear. In the latter cases it is convenient to discretize the global steps in smaller local steps. The algorithm presented in Fig. 6, that takes into account the use of double step, was employed for the resolution of the examples presented in the next section. Additional details can be found in Avon et al. (2002).

The routine that computes the stresses as a function of the generalized strains has to solve the local problem defined by the state law, the damage evolution laws, the yield functions of the hinges i and j and the normality rule. This system of nonlinear equations can be solved via the Newton method again. However, it is needed to know what damage variables and plastic multipliers are active at a given time. This

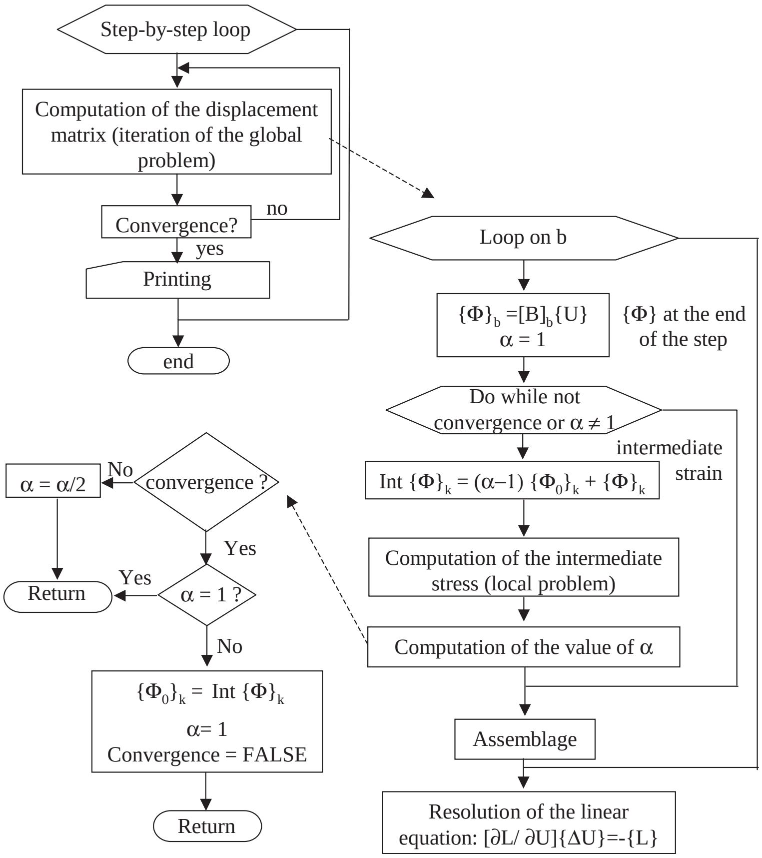

Fig. 6. Computational algorithm for the numerical resolution.

is done by using an elastic predictor, inelastic corrector and projection algorithm as the one described in Simo et al. (1988).

6. Examples

Bousias et al. (1995) carried out an experimental program on the behavior of reinforced concrete elements under biaxial bending and axial forces. The specimens consisted of square columns built as cantilever into a heavy foundation. The columns were subjected to axial forces and lateral displacements. In the first example of this section, the column was subjected to displacement-controlled lateral actions as the one shown in Fig. 7(a). The experimental results are shown in Fig. 7(b) and (d). The results of the numerical simulation are presented in Fig. 7(c) and (e). The data for the simulation consist in the interaction diagrams of the first cracking, yielding and ultimate moments, the interaction diagram of the ultimate plastic curvature (see Fig. 8), and the elastic stiffness coefficients (elastic modulus, equivalent inertia and area). Additionally, the geometry and the loading history must also be defined. The geometry was represented by only one finite element that was fixed (nil displacements) at one extreme while the other one was subjected to the imposed displacements indicated in Fig. 7(a). The axial forces were included as force-controlled loadings on the axis of the column.

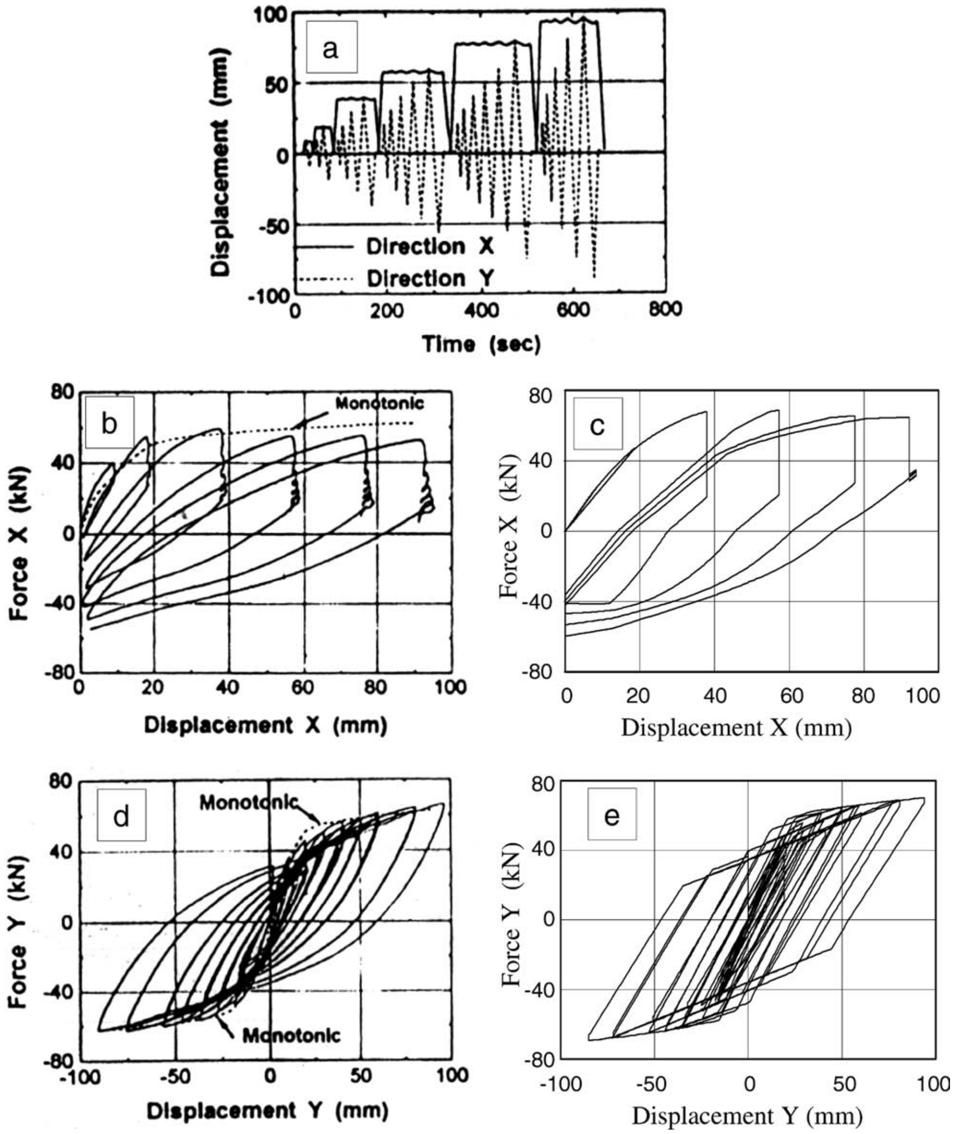

Fig. 7. Test by Bousias et al. (1995) and numerical simulation: (a) loading, (b) experimental hysteresis loops in the X global direction, (c) numerical simulation, (d) experimental hysteresis loops in the Y global direction and (e) numerical simulation.

The second example corresponds also to the numerical simulation of one of the tests described in Bousias et al. (1995). The loading can be described as follows. Circular deflection paths were imposed (Fig. 9(a)), consisting of four cycles at constant radius equal to about 20, 50, 80 and 110 mm. During the first two cycles, the axial load was kept low ($0.03A_c f_c'$) and was then increased to 0.15 $A_c f_c'$ during the last two levels. The experimental response is shown in Fig. 9(b) and (d). The results of the numerical simulation are represented in Fig. 9(c) and (e).

The last example consists in the numerical simulation of a test carried out at the University of California, Berkeley (Oliva, 1980; Oliva and Clough, 1987). The two-story framed structure represented in Fig. 10 was

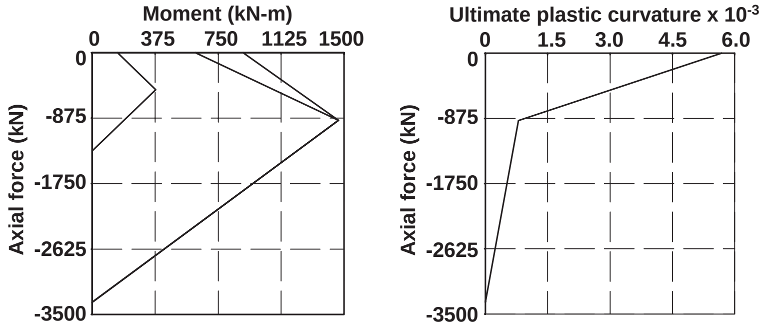

Fig. 8. Data for the simulation presented in Fig. 7.

subjected several times to ground displacements histories derived from the Taft earthquake record. The structure was placed with an angle of 25° with respect to the axis of the shaking table motion. Thus, the structure was subjected to biaxial solicitations. First two low intensity shakes, with peak acceleration amplitude of 0.06 g, were applied on the structure. The goal of this loading was to induce minor cracking in the virgin frame to replicate the condition of a real structure which has been under service loading. Then the structure was subjected to a severe shaking with a peak acceleration of 0.685 g.

The beams and columns of the structure were represented by the finite element described in this paper (one element for frame member). Standard elastic plate elements (Kirchoff theory) were used to represent the first and second floor slabs. The entire structure was represented by 16 inelastic elements (beams and columns) and eight elastic elements (slabs). The data for the numerical simulation was obtained from the cross section properties of the frame indicated in Oliva (1980). The loading input for the analysis consisted in the actual Taft earthquake record obtained from the PEER strong motion data base (web version). The real loading applied on the structure, although derived from this record, is not exactly the same and some differences can be appreciated between both records.

The damage state after the first low intensity shake obtained with the computer program can be seen in Fig. 11(a). In the figure, the maximum values of damage of each hinge (there are four values per hinge) are shown. It can be noticed that no value exceeded 0.26. This corresponds indeed to minor cracking that does not need reparation, as indicated in Alarcón et al. (2001).

The numerical results obtained after the severe shake are shown in Figs. 11(b), 12 and 13. In Fig. 12, the experimental and computed displacements of the first floor are presented (there is no information of the second floor displacements in Oliva, 1980). Fig. 13 shows the computed and observed local behavior of one of the columns of the first floor. Fig. 11(b) indicates the final state of damage after the numerical simulation. Very high values of damage can be observed in this figure. Structures with values of damage that high should not, after conventional engineering criteria, be repaired (see Alarcón et al., 2001). The structure was nonetheless repaired and tested again (Oliva, 1980), and its behavior does not seem bad.

7. Limitations of the proposed model

The limitations of the model can be grouped into two different categories. The first one includes the restrictions due to the general framework of the model, i.e. lumped damage mechanics. It is clear that the model may be used only in the situations where cracking and plasticity are limited to restricted areas of the frame member. Often, this is the case under earthquake loadings, but if extensive cracking of variable density develops across the elements, fiber beam theories are more adequate. On the other hand, fiber beam

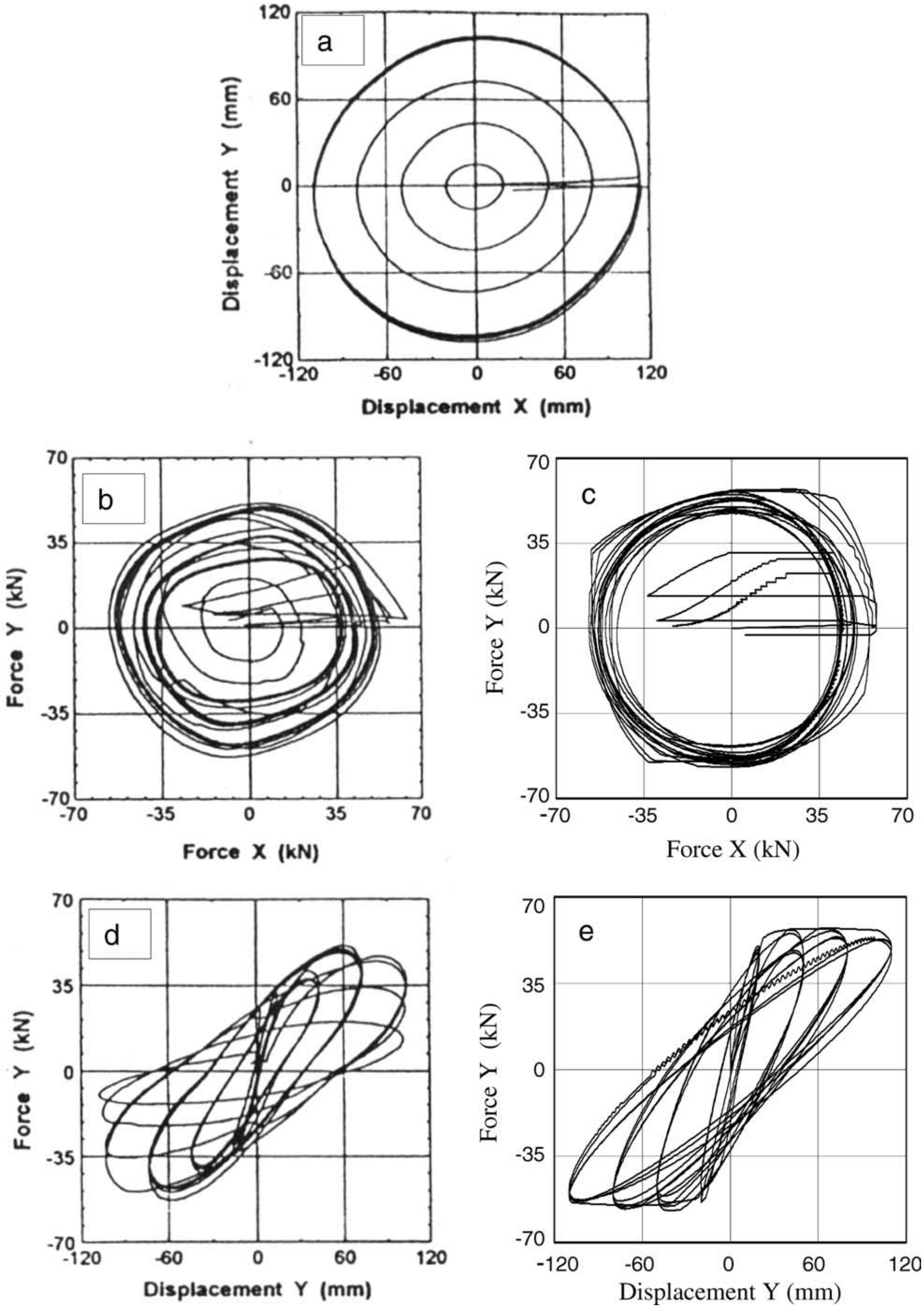

Fig. 9. Test by Bousias et al. (1995) and numerical simulation: (a) loading, (b) measured force path, (c) numerical simulation, (d) experimental hysteresis loops in the Y global direction and (e) numerical simulation.

models need the use of some regularization scheme since mathematical problems due to localization may arise in that case.

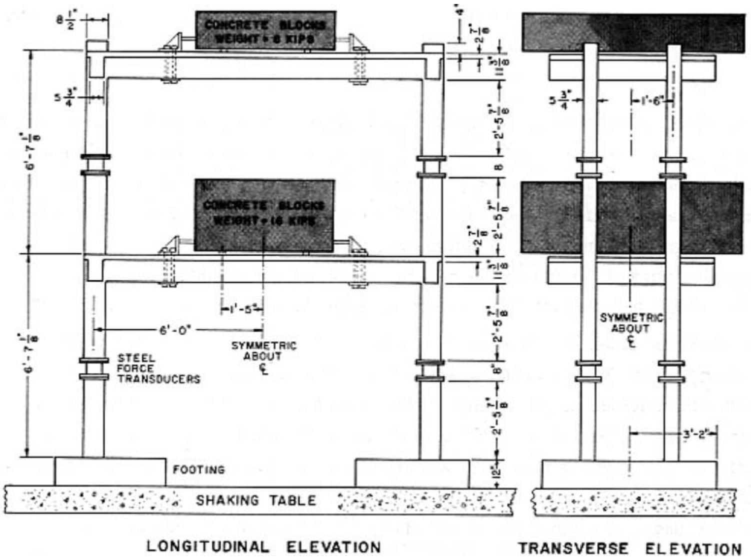

Fig. 10. Framed structure subjected to earthquake loading after Oliva (1980).

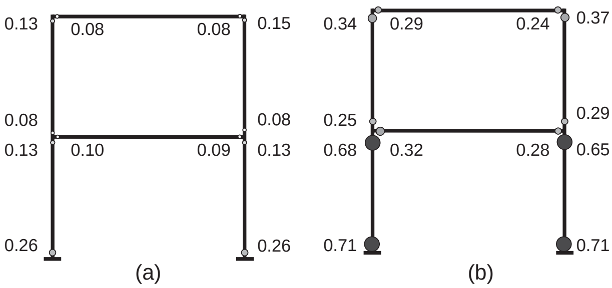

Fig. 11. Damage distribution in the frame: (a) after a low intensity shake and (b) after a severe shake.

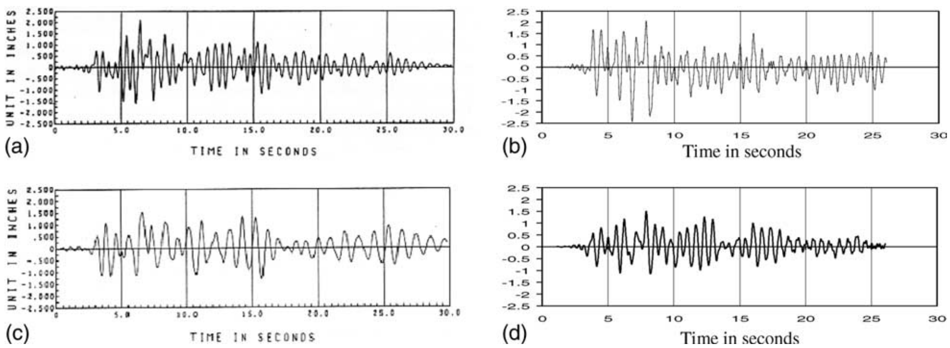

Fig. 12. Displacements of the first floor: (a) longitudinal axes, experimental; (b) longitudinal axes, computed; (c) transversal axes, experimental and (d) transversal axes, computed.

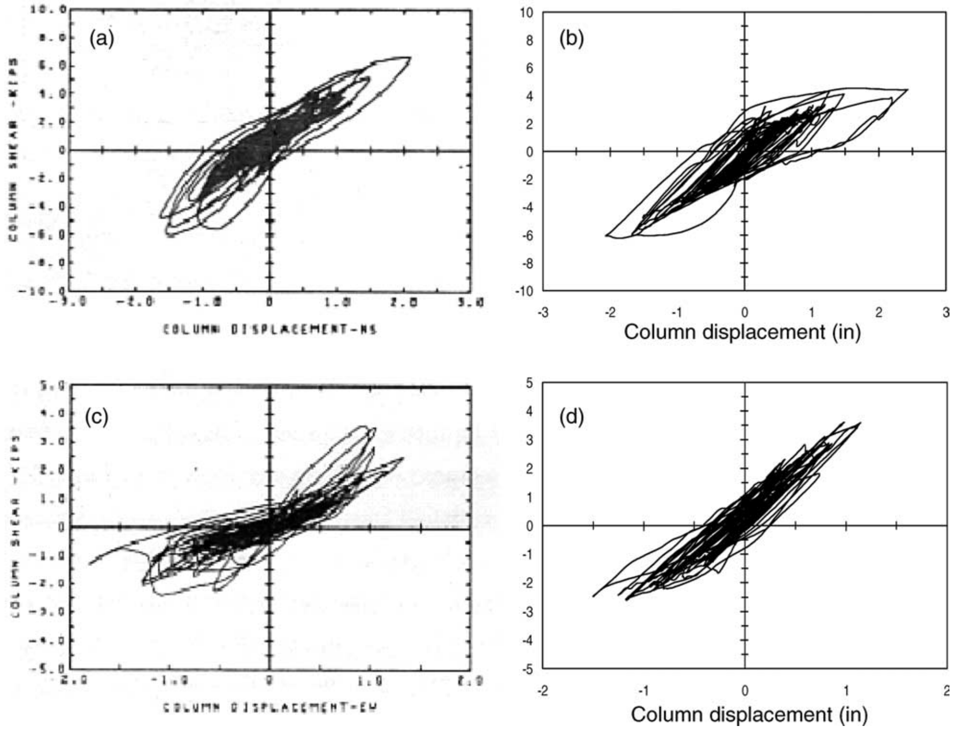

Fig. 13. Local behavior of a first floor column: (a) longitudinal axes, experimental; (b) longitudinal axes, computed; (c) transversal axes, experimental and (d) transversal axes, computed.

Localization is not a problem in lumped damage mechanics. In fact, lumped damage mechanics can be seen as a regularization procedure valid in the case of beams. The use of regularization methods in strain-softening problems involves the introduction of "internal length scales" that are essential for the proper description of the localization phenomenon. The concept of "internal length scale" in lumped damage mechanics can be related with two characteristics of the method. In the first place, the fact that energy dissipation is assumed to be lumped into locations of zero length: the inelastic hinges. It must be emphasized that although concentrated into zero length zones, this energy dissipation is no nil. The second aspect related with the internal length scale, is the fact that the user defines the number and locations of the inelastic hinge when she or he discretizes the structure into finite elements. The structural response is independent of the number or the size of the finite elements as far as the position of the inelastic hinges is not changed. Therefore, this constitutes a second limitation of the model: the position of the inelastic hinges is part of the data of the problem. However, in most earthquake engineering applications, this is not an important restriction since the user has almost always an accurate idea of where these inelastic hinges can appear, as it can be seen, for instance, in the last example of Section 6.

The second category of limitations is related with the model itself. For instance, the damage evolution law (13) becomes a sort of Griffith criterion in the case of monotonic loadings in only one plane. The Griffith criterion does not take into account low or high cycle fatigue effects and therefore, neither does the proposed model. In this sense, the model could be improved by the use of generalized forms of the Griffith criteria that are available in the continuum damage and fracture mechanics literature. Additionally, a Rankine-like criterion of damage for general biaxial loadings was assumed. This criterion has the merit of simplicity but implies an uncoupling between crack evolution in the different faces of the element. The limitations of this hypothesis have not been evaluated yet and, probably, better criteria are possible without implying a significant increment in the complexity of the model.

The model only includes damage variables related to flexural effects. The influence that torsion and axial forces may have on the flexural behavior has been taken into account in a simplified way. However, no specific torsion or axial related damage has been introduced. Therefore the model should be used only in cases of flexural dominated behavior.

Some discrepancies can be appreciated between model and test in the examples of the previous section, for instance in the case of Fig. 9. In addition to the aforementioned limitations of the model, it must be added the uncertainty of the data for the simulation. Specifically that related with the variation of the axial loads during the test. The interaction diagrams needed for the simulation (see Fig. 8) also introduce some uncertainty since these properties can be computed with errors that are almost always larger than 10% or 15%. The same remarks apply for the last example of Section 6. For instance, the data related with the earthquake records that are used as input are not the same for test and simulation. On the other hand the results of the first example of Section 6 (Fig. 7) were excellent.

8. Final remarks and conclusions

The common philosophy of most codes (perhaps all of them) for building design is that buildings should be able to resist minor earthquakes without damage, to resist moderate earthquake with some, repairable, damage and to resist mayor earthquakes without collapse, although with damage that might be non-repairable. It is then obvious that quantitative damage analysis of building structures should be a mayor subject of the structural engineering.

However, in practice, damage analyses are not very often carried out. In the cases where such analyses are needed, the common procedure consists in inelastic analyses based on plasticity theories. However, these results are of little use for the practical engineer. Thus, a damage analysis is then accomplished via semi-empirical rules by post-processing. The coupled damage/structural analysis of the structure constitutes a more rational approach to the problem. Concepts from fracture mechanics and continuum damage mechanics can be adapted to the analysis of framed structures with plastic hinges and the resulting theory constitutes a good compromise between simplicity, rationality and accuracy, for engineering purposes, even for complex three-dimensional problems.

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References

Aktan, A., Pecknold, D., Sozen, M., 1973. Effects of two-dimensional earthquake motion on a reinforced concrete column. Civil Engineering Studies, Structural Research Series Report No. 399, University of Illinois, Urbana, I11.

Alarcón, E., Recuero, A., Perera, R., Lopez, C., Gutierrez, J.P., De Diego, A., Picon, R., Florez Lopez, J., 2001. A reparability index for reinforced concrete members based on fracture mechanics. Engng Struct. 23 (6), 687–697.

Anthoine, A., Guedes, J., Pegon, P., 1997. Non-linear behavior of reinforced concrete beams: from 3D continuum to 1D member modeling. Comput. Struct. 65 (6), 949–963.

Avon, D., Marante, M.E., Flórez-López, J., 2002. El doble paso de integración: un algoritmo computacional para mejorar la convergencia en problemas altamente no lineales (in Spanish). Desarrollos recientes en Métodos Numéricos para Ingeniería y Ciencias Aplicadas, Proceedings of CIMENICS 2002, Caracas, Venezuelai.

Bahn, B.Y., Hsu, C.T.T., 2000. Cyclically and biaxially loaded reinforced concrete slender columns. ACI Struct. J. 97 (3), 444–454.

Bolzon, G., 1996. An approximate method for fatigue-life prediction of framed structures. Fatigue Fract. Engng Mater. Struct. 19 (12), 1481–1491.

Bousias, S., Verzeletti, G., Fardis, M., Gutierrez, E., 1995. Load-path effects in columns under biaxial bending with axial forces. J. Engng Mech. ASCE 121 (5), 596–605.

Bresler, B., 1960. Design criteria for reinforced concrete columns under axial load and biaxial bending. J. ACI, 57.

Chen, P., Powell, G., 1982. Generalized plastic hinge concepts for 3-D beam column elements. EERC 82-20, Earthquake Engineering Research Center, University of California, Berkeley, California.

Cipollina, A., López-Inojosa, A., Flórez-López, J., 1995. A simplified damage mechanics approach to nonlinear analysis of frames. Comput. Struct. 54 (6), 1113–1126.

Flórez-López, J., 1998. Frame analysis and continuum damage mechanics. J. Eur. Mech. 17 (2), 269–284.

Lai, S., Will, G., Otani, S., 1984. Model for inelastic biaxial bending of concrete members. J. Struct. Engng ASCE 110 (11).

Marante, M.E., Flórez-López, J., 2002. Model of damage for RC elements subjected to biaxial bending. Engng Struct. 24 (9), 1141–1152.

Mazza, F., 1998. Modelli di danneggiamento nell'analisi sismica non lineare di strutture intelaiate in C.A. (in Italian). Tesis, Università degli Studi della Calabria Cosenza, Italy.

Nigam, N., 1967. Inelastic interactions in the dynamic response of structures. WWRL 67-64, Earthquake Engineering Research Laboratory, California Institute of Technology, Pasadena, California.

Okada, T., Sehi, M., Asai, S., 1976. Response of reinforced concrete columns to bidirectional horizontal force and constant axial force. Bulletin of Earthquake Research Center, No. 10, University of Tokio, Tokyo, Japan.

Oliva, M.G., 1980. Shaking table testing of a reinforced concrete frame with biaxial response. Report No. EERC 80-28, Earthquake Engineering Research Center, University of California, Berkeley, California.

Oliva, M.G., Clough, R.W., 1987. Biaxial seismic response of R/C frames. J. Struct. Engng ASCE 113 (6), 1264–1281.

Oller, S., Barbat, A., Oñate, E., Hanganu, A., 1992. A damage model for the seismic analysis of building structures. In: Proceedings Tenth World Conference on Earthquake Engineering, Madrid, Balkema, Rotterdam, vol. 5, pp. 2593–2598.

Padilla-Mora, R., Schnobrich, W., 1974. Non-linear response of framed structures to two-dimensional earthquake motion. Civil Engineering Studies, Research Series Report No. 408, University of Illinois, Urbana, I11.

Perdomo, M.-E., Ramirez, A., Flórez-López, J., 1999. Simulation of damage in RC frames with variable axial forces. Earthquake Engng Struct. Dyn. 28 (3), 311–328.

Perera, R., Carnicero, A., Alarcón, E., Gómez, S., 2000. A fatigue damage model for seismic response of RC structures. Comput. Struct. 78, 293–302.

Roufaiel, M., Meyer, C., 1987. Analytical modeling of hysteretic behavior of R/C frames. J. Struct. Div. ASCE 113 (3), 429–443.

Selna, L., Lawder, J., 1977. Biaxial inelastic frames. ACI Special Pub. SP-53, ACI, Detroit, Mich., pp. 440–461.

Simo, J., Kennedy, A., Govindjee, S., 1988. Non-smooth multisurface plasticity and viscoelasticity. Loading/unloading conditions and numerical algorithms. Int. J. Numer. Meth. Engng 26, 2161–2185.

Takizawa, H., Aoyama, H., 1976. Biaxial effects in modeling earthquake response of R/C structures. Earthquake Engng Struct. Dyn. 4, 523–552.

Zeris, C., Mahin, S., 1991. Behavior of reinforced concrete structures subjected to biaxial excitation. J. Struct. Engng 117 (9), 2657–2673.